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ON A MAX-MIN PROBLEM CONCERNING WEIGHTS OF EDGES

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The weight $w(e)$ of an edge $e=uv$ of a graph is defined to be the sum of degrees of the vertices u and v. In 1990 P. Erdős asked the question: What is the minimum weight of an edge of a graph G having n vertices and m edges? This paper brings a precise answer to the above question of Erdős.

1. Introduction

The weight $w(e)$ of an edge $e=uv$ of a graph G is defined to be the sum of degrees of the vertices u,v. This concept of the weight of an edge was introduced by Kotzig [8] who proved the following beautiful result: Every planar 3-connected graph contains an edge of weight not exceeding 13.

This result was further developed in various directions. Grünbaum [4], Jucovič [7], Borodin [1], Fabrici and Jendrol' [3] have studied inequalities for the number of edges having weight not exceeding 13 in planar 3-connected graphs. Ivančo [5] has found an analogue of Kotzig's result for graphs with minimum degree at least 3 and embedded on orientable 2-manifolds. The analogue of Kotzig's result for triangulations of orientable 2-manifolds can be found in Zaks [9].

Recently Fabrici and Jendrol' [3] proved that each 3-connected planar graph of maximum degree $\geq k$ contains a path on k vertices such that each of its vertices has degree at most $5k$; the bound $5k$ being best possible. Enomoto and Ota [2] have proved that each planar 3-connected graph of

Mathematics Subject Classification (2000): 05C35 order at least k contains a connected subgraph on k vertices such that the degree sum of the vertices of this subgraph is at most $8k-1$.

At the Fourth Czechoslovak Symposium on Combinatorics held in Prachatice, Erdős asked the question: What is the minimum weight of an edge e of a graph G having n vertices and m edges? Denote this integer by $W(n,m)$. More precisely, for integers $n,m,n\geq 2, 0\leq m\leq \binom{n}{2}$ let $\mathcal{G}(n,m)$ be the family of all graphs having n vertices and m edges. Then

$$W(n,m) = \max_{G\in\mathcal{G}(\mathrm{n,m})}\{\min_{e\in E(G)} w(e)\}$$

In [6] Ivančo and Jendrol' proved some partial results. They observed that the weight of any edge e of a graph $G\in\mathcal{G}(n,m)$ cannot be larger than $m+1$. Because of the graph $K_{1,m}\cup\overline{K}_{n-m-1}$ we have

$$W(n,m) = m+1 \text{ for every } 1\leq m< n. \tag{1}$$

For $m=\binom{n}{2}-r$ with $0\leq r<n-1$ the following theorem of Ivančo and Jendrol' [6] gives the corresponding values for $W(n,m)$.

Theorem 1.1. *Let $m=\binom{n}{2}-r$ with $0\leq r<n-1$. Then*
(i) *$W(n,m)=2n-2$ for $r=0$ and $W(n,m)=2n-3$ for $r=1$;*
(ii) *$W(n,m)=2n-4$ for $2\leq r\leq\lfloor\frac{n}{2}\rfloor$ or $r=3$,*
(iii) *$W(n,m)=2n-5$ for $\lfloor\frac{n}{2}\rfloor<r\leq\lceil\frac{n+2}{2}\rceil$ or $r=6$;*
(iv) *$W(n,m)=2n-6$ in all other cases.*

Graphs achieving this value can be obtained by taking a K_n and removing r independent edges or edges of a triangle $(r=3)$ in cases (i) and (ii). In case (iii) take K_n and remove r–3 independent edges and edges of an independent triangle or edges of a $K_4(r=6)$. Finally, in case (iv) edges of a cycle of length r are deleted from K_n.

Theorem 1.2 **([6]).** *Let $a=\lceil\frac{1}{2}(1+\sqrt{1+8m})\rceil$ and $b=\frac{1}{2}(a^2-a-2m)$, let $h=\left\lceil\frac{1}{2}(2n-1-\sqrt{(2n-1)^2-8m})\right\rceil$ and let p,k be integers such that $hk+p=m$, $h+k\leq n$ and $h(h-3)<2p\leq h(h-1)$. Let $f(n,m)=h+k+\lfloor\frac{2p}{h}\rfloor$ and $g(n,m)$ be defined as follows*

$$g(n,m)=\begin{cases} 2a-2 \text{ if } b=0;\\ 2a-3 \text{ if } b=1;\\ 2a-4 \text{ if } 2\leq b\leq\lfloor\frac{a}{2}\rfloor \text{ or } b=3;\\ 2a-5 \text{ if } \lfloor\frac{a}{2}\rfloor<b\leq\lceil\frac{a+2}{2}\rceil \text{ or } a=8 \text{ and } b=6;\\ 2a-6 \text{ in all other cases.}\end{cases}$$

Then

$$W(n,m) \geq \max\{f(n,m), g(n,m)\}.$$

Ivančo and Jendrol' [6] posed the following conjecture which we are going to prove in the next section.

Conjecture 1.3.

$$W(n,m) = \max\{f(n,m), g(n,m)\}.$$

2. Proof of the conjecture

Observe that $a = \min\{t \in I\!N \,|\, \binom{t}{2} \geq m\}$. Thus $b \leq a-2$. Hence, if G consists of a component with a vertices and $n-a$ isolated vertices, then $g(n,m)$ is maximum by Theorem 1.1. Therefore we may assume that any graph G^* with a larger minimum weight than G ($w^* > w = g(n,m)$, where w^* and w denote the minimum weight in G^* and in G, respectively) has at least $a+1$ vertices of positive degree. Further we can assume $a \geq 4$ by (1), Theorem 1.1 and Theorem 1.2.

So let $G^* = (V,E)$ be a graph with larger minimum weight than G and let $G^* \in \mathcal{G}(n,m)$. Let $I := \{v \in V \,|\, d(v) = 0\}$ (isolated vertices), $R := \{v \in V \backslash I \,|\, d(v) \leq \frac{w}{2}\}$, $S := N(R) \backslash R$ and $T := V \backslash (I \cup R \cup S)$, where $N(R)$ denotes the set of vertices having neighbours in R.

Then by the assumption $G^*[R]$ is independent.

Proposition 2.1. $R \neq \emptyset$

Proof. Suppose $R = \emptyset$. Then $2m \geq \frac{w+1}{2} \cdot (a+1)$. We distinguish five cases. each depending on the value of b.

Case 1. If $w = 2a-2$, $b = 0$, then G has $2m = \frac{a(a-1)}{2}$ edges and G^* has $2m \leq \frac{2a-1}{4}(a+1)$ edges. This implies $a(a-1) \geq \left(a - \frac{1}{2}\right)(a+1)$, a contradiction.

Analogously we proceed in all other cases.

Case 2. If $w = 2a-3, b = 1$, then $a(a-1)-2 \geq (a-1)(a+1)$, a contradiction.

Case 3. If $w = 2a-4, 2 \leq b \leq \lfloor \frac{a}{2} \rfloor$ or $b = 3$ then $a(a-1) - 2b = a^2 - a - 2b \geq \left(a - \frac{3}{2}\right)(a+1) = a^2 - \frac{a}{2} - \frac{3}{2}$, a contradiction.

Case 4. If $w = 2a-5$, $\lfloor \frac{a}{2} \rfloor < b \leq \lceil \frac{a+2}{2} \rceil$ or $a = 8, b = 6$ then $a(a-1) - 2b = a^2 - a - 2b \geq (a-2)(a+1) = a^2 - a - 2$, a contradiction.

Case 5. If $w = 2a–6$ then $a(a–1)–2b = a^2–a–2b \geq \left(a - \frac{5}{2}\right)(a+1) = a^2 – \frac{3}{2}a – \frac{5}{2}$, a contradiction since $2b > 2\lceil \frac{a+2}{2} \rceil \geq a+2$. ∎

Proposition 2.2. $T = \emptyset$.

Proof. Suppose $T \neq \emptyset$. Then $d(v) \geq \left\lceil \frac{w+1}{2} \right\rceil$ for all $v \in S \cup T$ and $N(T) \subseteq S \cup T$. This implies $|S \cup T| \geq \left\lceil \frac{w+3}{2} \right\rceil$. We now estimate the degree-sums for the vertices of $S \cup T$. From above we obtain $2m \geq \sum_{v \in S \cup T} d(v) \geq \left\lceil \frac{w+3}{2} \right\rceil \left\lceil \frac{w+1}{2} \right\rceil$.

Analogously as in the previous proposition we distinguish five cases:

Case 1. If $w = 2a-2, b = 0$ then $a(a-1) \geq \sum_{v \in S \cup T} d(x) \geq \left\lceil \frac{w+3}{2} \right\rceil \lceil \frac{w+1}{2} \rceil \geq (a+1)a$, a contradiction.

Case 2. If $w=2a-3, b=1$ then $a(a-1)-2 \geq a(a-1)$, a contradiction.

Case 3. If $w=2a-4$, $2 \leq b \leq \lfloor \frac{a}{2} \rfloor$ or $b=3$, then $a(a-1)-2b = a^2-a-2b \geq a(a-1) = a^2-a$, a contradiction.

Next observe that there exists an edge uv with $u \in R, v \in S$ such that $d(u)+d(v) \geq w+1$.

Case 4. Let $w=2a-5$, $\lfloor \frac{a}{2} \rfloor < b \leq \lceil \frac{a+2}{2} \rceil$ or $a=8, b=6$. Then $a(a-1)-2b = a^2-a-2b = 2m \geq \sum_{x \in S \cup T \setminus \{v\}} d(x)+d(u)+d(v) \geq (\left\lceil \frac{w+3}{2} \right\rceil - 1)\left\lceil \frac{w+1}{2} \right\rceil + d(u)+d(v) \geq ((a-1)-1)(a-2)+(2a-4) = a^2-2a$, a contradiction since $b > \frac{a}{2}$.

Case 5. Let $w=2a-6$. Analogously as in the previous case we get $a(a-1)-2b = a^2-a-2b \geq ((a-1)-1)(a-2)+(2a-5) = a^2-2a-1$, a contradiction since $2b > 2\left\lceil \frac{a+2}{2} \right\rceil \geq a+2$. ∎

Hence we may assume from now on that $V(G^*) = R \cup S \cup I$. Let $r := |R|$ and $s := |S|$. By the assumption $d(v) \leq \frac{w}{2}$ for all $v \in R$ and $d(v) \geq \frac{w+1}{2}$ for all v $\in S$ we conclude that $r \geq 3$. For given n, m with $m = \binom{s}{2} + s \cdot r - q, 0 \leq q < s$, let $\mathcal{G}(r,s,q) \subseteq \mathcal{G}(n,m)$ be the class of all graphs such that $G[R]$ is independent and let

$$W(r,s,q) = \max_{G \in \mathcal{G}(\text{r,s,q})} \{ \min_{e \in E(G)} w(e) \}$$

Proposition 2.3.

$$W(r,s,q) = \begin{cases} 2s+r-1, \, q=0 \\ 2s+r-2, \, 1 \leq q \leq \frac{s}{2} \\ 2s+r-3, \, \frac{s}{2} < q < s \end{cases}$$

Graphs achieving this value can be obtained by taking a K_s and removing q independent edges $(K_s - qK_2)$ or q edges of a cycle $(K_s - C_q)$, respectively.

Proof. Suppose there exists a graph $G \in \mathcal{G}(r,s,q)$ with a minimum weight w^* larger than $W(r,s,q)$. Since $G[R]$ is independent we have

$$m = \sum_{v \in S} d_R(v) + \frac{1}{2}\sum_{v \in S} d_S(v) = \sum_{v \in S} d(v) - \frac{1}{2}\sum_{v \in S} d_S(v) \geq \sum_{v \in S} d(v) - \binom{s}{2}.$$

Here $d_X(v)$ denotes the number of neighbours of the vertex v in the set X.

Case $q=0$.
Then $w^* \geq 2s+r$. Let uv be an edge of G with $u\in R, v\in S$ and having the weight w^*. Then $d(u)+d(v)=w(uv)\geq 2s+r$. Now $d(u)=d_S(u)\leq s$ implies $d(v)\geq (2s+r)-s=s+r$ for all $v\in S$.
Thus $\sum_{v\in S} d(v)\geq s(s+r)$ and $m\geq s(s+r)-\binom{s}{2}=s\cdot r+\binom{s+1}{2}>s\cdot r+\binom{s}{2}=m$, a contradiction.

Case $1\leq q\leq \frac{s}{2}$.
Then $w^*\geq 2s+r-1$. This time $d(v)\geq (2s+r-1)-s=s+r-1$ for all $v\in S$. Thus $\sum_{v\in S} d(v)\geq s(s+r-1)$ and $m\geq s(s+r-1)-\binom{s}{2}=s\cdot r+\binom{s}{2}>s\cdot r+\binom{s}{2}-q=m$, a contradiction, since $q\geq 1$.

Case $\frac{s}{2}<q<s$.
Then $w^*\geq 2s+r-2$. Suppose $G[R\cup S]$ is not complete. Then there is a vertex $u\in R$ with $d_s(u)=s-k$ for some $1\leq k\leq s-1$ (since $N_S(u)\neq\emptyset$ for all $u\in R$) implying $s\geq 2$.

Then $d(v)\geq (2s+r-2)-(s-k)$ for all $v\in N_S(u)$ and

$$\begin{aligned}\sum_{v\in S} d(v) &= \sum_{v\in N_S(u)} d(v) + \sum_{v\in S\setminus N_S(u)} d(v)\\ &\geq (s-k)(s+r+k-2)+k(s+r-2)\\ &= s(s+r-2)+k(s-k)\geq s(s+r-2)+s-1.\end{aligned}$$

Thus

$$\begin{aligned}m &\geq s(s+r-1)-1-\binom{s}{2}\\ &= s\cdot r+\binom{s}{2}-1>s\cdot r+\binom{s}{2}-q=m,\end{aligned}$$

a contradiction, since $q>1$. ∎

For given m,n we will now compare the weights $w(G(r,s,q))$ for all possible triples r,s,q.

Case 1. Let $F(r,s,q)$ be a graph with $w(F(r,s,q))=f(n,m)$. Note that s is minimum in this case. We now examine for which values of t we have $w(F(r,s,q))\geq w(G(r-2t,s+t,q'))$. Depending on q we consider the inequality

$$w(F(r,s,q))\leq w(G(r-2t,s+t,0)).$$

Subcase 1.1. $q=0$. We have $w(F(r,s,0))=2s+r-1\geq w(G(r-2t,s+t,q'))$ for arbitrary q' and consider the inequality

$$sr+\binom{s}{2}\leq(s+t)(r-2t)+\binom{s+t}{2}$$
$$\iff t(3t+2s+1-2r)\leq 0.$$

Since $t>0$ this implies

$$3t+2s+1-2r\leq 0$$
$$\iff t\leq\frac{2r-2s-1}{3}.$$

Subcase 1.2. $1\leq q\leq\frac{s}{2}$.
We have $w(F(r,s,q))=2s+r-2\geq w(G(r-2t-1,s+t,q'))$ for arbitrary q' and consider the inequality

$$sr+\binom{s}{2}-1\leq(s+t)(r-2t-1)+\binom{s+t}{2}$$
$$\iff t(3t+2s+3-2r)+2s-2\leq 0.$$

Since $t>0$ this implies

$$3t+2s+3-2r\leq 0$$
$$\iff t\leq\frac{2r-2s-3}{3}.$$

Note that any "solution" for t and $q=1$ remains a "solution" for t and $1\leq q\leq\frac{s}{2}$.
Subcase 1.3. $\frac{s}{2}<q<s$.
We have $w(F(r,s,q))=2s+r-3\geq w(G(r-2t-2,s+t,q'))$ for arbitrary q' and consider the inequality

$$sr+\binom{s}{2}-\frac{s+1}{2}\leq(s+t)(r-2t-2)+\binom{s+t}{2}$$
$$\iff t(3t+2s+5-2r)+4s-2\frac{s+1}{2}\leq 0.$$

Since $t>0$ and $s\geq 1$ this implies

$$3t+2s+5-2r\leq 0$$
$$\iff t<\frac{2r-2s-5}{3}.$$

Note that any "solution" for t and $q=\lceil\frac{s+1}{2}\rceil$ remains a "solution" for t and $\frac{s}{2}<q<s$. For all three cases considered above any t satisfying $1\leq$

$t < \frac{2r-2s-5}{3}$ is a common solution. Moreover, this implies $r-s \geq 5$. Hence for all r,s,q with $r-s\geq 5$ we have $w(G(r,s,q)) \geq w(G(r',s+1,q'))$, where $rs+\binom{s}{2}-q=r'(s+1)+\binom{s+1}{2}-q'$.

Now for s increasing r is non-increasing. Hence, if $w(G(r',s',q')) > w(F(r,s,q))$ then $r'-s'\leq r-s\leq 4$.

Case 2. Let $H(a,b)$ be a graph with $w(H(a,b))=g(n,m)$. For convenience we set $a=s, b=q$ and examine for which values of t we have $w(G(r',s–t,q'))\leq w(H(s,q))$. Note that s is maximum in this case.

Subcase 2.1. $0\leq q\leq 1$.
We consider the inequality $\binom{s}{2}-q\leq(s-t)(2t-1)+\binom{s-t}{2}-q$. Then $w(H(s,q))=2s-2-q=w(G(2t-1,s-t,q'))$ for $q'\geq q$. Moreover, for $r'\leq 2t-2$ we have $w(G(r',s-t,q'))\leq 2(s-t)+r'-1\leq 2s-3\leq 2s-2-q$ for any $q'\geq 0$.

Subcase 2.2. $2\leq q\leq 3$.
We consider the inequality $\binom{s}{2}-q\leq(s-t)(2t-2)+\binom{s-t}{2}-1$ for $2\leq q\leq \frac{s}{2}$ or $q=3$ (and $s=5$). Then $w(H(s,q))=2s–4\geq w(G(2t–2,s–t,q'))$ for any $q'\geq 1$. Moreover, for $r'\leq 2t-3$ we have $w(G(r',s-t,q'))\leq 2(s-t)+r'-1\leq 2s-4$ for any $q'\geq 0$.

Subcase 2.3. $4\leq q\leq 6$.
We consider the inequality $\binom{s}{2}-q\leq(s-t)(2t-3)+\binom{s-t}{2}-1$ Since $q\leq s-2$ we have $q\leq\left\lceil\frac{s+2}{2}\right\rceil$ for $q=4$ or $q=5$. For $q=6$ we either have $q\leq\lceil\frac{s+2}{2}\rceil$ for $s\geq 9$ or $s=8, q=6$. Then $w(H(s,q))=2s-5\geq w(G(2t-3,s-t,q'))$ for any $q'\geq 1$. Moreover, for $r'\leq 2t-4$ we have $w(G(r',s-t,q'))\leq 2(s-t)+r'-1\leq 2s-5$ for any $q'\geq 0$.

Subcase 2.4. $q\geq 7$.
We consider the inequality $\binom{s}{2}-q\leq(s-t)(2t-4)+\binom{s-t}{2}-1$. Then $w(H(s,q))\geq 2s-6\geq w(G(2t-4,s-t,q'))$ for any $q'\geq 1$. Moreover, for $r'\leq 2t-5$ we have $w(G(r',s-t,q'))\leq 2(s-t)+r'-1\leq 2s-6$ for any $q'\geq 0$.

The four inequalities from above can be written as $\binom{s}{2}-q\leq(s-t)(2t-k)+\binom{s-t}{2}-\min(1,q)$ for $k=1,2,3,4 \Longleftrightarrow t(3t-2s-2k-1)+2(sk-q+\min(1,q))\geq 0$. Since $t>0$ and $sk-q+\min(1,q)>0$ we have $3t-2s-2k-1<0 \Longleftrightarrow t<\frac{2s+2k+1}{3}$.

Suppose now that for some $t\geq\frac{2s+2k+1}{3}$ we have $\binom{s}{2}-q\geq(s–t)(2t–k)+\binom{s-t}{2}$. If $w(G(r',s-t,q'))>w(H(q,s))$, then we must have $r'\geq 2t-k$. Therefore, $r'-s'\geq(2t-k)-(s-t)=3t-s-k\geq s+k+1\geq 6$ (since $s=a\geq 4$). Hence, if $w(G(r',s',q'))>w(H(q,s))$ then $r'-s'\geq 6$.

Since r',s' cannot satisfy both $r'-s'\leq 4$ (Case 1) and $r'-s'\geq 6$ (Case 2) we conclude that $W(n,m)=\max\{w(F(q,r,s)),w(H(q,s))\}=\max\{f(n,m),g(n,m)\}$. This completes the proof of the conjecture. ∎

3. The values of W(n,m)

Following the proof of the conjecture we have proved the following proposition for two constants $c_1 = 37$ and $c_2 = 9$.

Proposition 3.1. *There are two constants $c_1, c_2 > 0$ such that*

$$W(n,m) = f(n,m) \text{ for all } m \text{ with } 1 \le m \le \frac{9n^2 - c_1 n}{50},$$

$$W(n,m) = g(n,m) \text{ for all } m \text{ with } \binom{n}{2} \ge m \ge \frac{9n^2 + c_2 n}{50}.$$

With Theorem 1.1 and Theorem 1.2 this implies the following corollary.

Corollary 3.2.

$$W(n,m) = f(n,m) \text{ for all } m \text{ with } 1 \le m \le \max\left(n-1, \frac{9n^2 - c_1 n}{50}\right),$$

$$W(n,m) = g(n,m) \text{ for all } m \text{ with } \binom{n}{2} \ge$$

$$m \ge \min\left(\binom{n}{2} - n + 2, \frac{9n^2 + c_2 n}{50}\right).$$

For the remaining values of m (for given n) a frequent change of $f(n,m)$ or $g(n,m)$ being maximum can be observed. Hence, for growing n and $1 \le m \le \max\left(n-1, \frac{9n^2 - c_1 n}{50}\right)$, the graph of $W(n,m)$ has a sawtooth-shape. For $1 \le m \le 2n-3$ we obtain

$$W(n,m) = \begin{cases} m+1, 1 \le m \le n-1 \\ \left\lfloor \frac{m+5}{2} \right\rfloor, n \le m \le 2n-3 \end{cases}$$

Obviously, $W(n,n-1) - W(n,n) = n - \left\lfloor \frac{n+5}{2} \right\rfloor = \lfloor \frac{n-4}{2} \rfloor$. Further evaluation of $W(n,m)$ shows further jumps.

Finally, by analysing $f(n,m)$ and $g(n,m)$, one can show that for each pair of n,k with $n \ge 2$ and $2 \le k \le 2n-2$ there exists m with $1 \le m \le \binom{n}{2}$ such that $W(n,m) = k$.

Observe that $f(n,m+1) - f(n,m) \le 1$ and $g(n,m+1) - g(n,m) \le 1$ for all $n \ge 2$ and $1 \le m \le \binom{n}{2} - 1$. Hence, with $f(n,1) = g(n,1) = 2$ and $f(n,\binom{n}{2}) = g(n,\binom{n}{2}) = 2n-2$, for both $f(n,m)$ and $g(n,m)$ all values of k for $2 \le k \le 2n-2$ are achieved for suitable graphs.

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